Counting to One Million

When they read in David Schwartz's book *How Much Is a Million?* that it takes 23 days to count to one million, many students refuse to believe it. When they exhibit this skepticism of data, or try to confirm it, they are demonstrating an important attitude in mathematics. Living as they do in an age in which they are inundated with numerical information, students who challenge data become critical consumers of facts and figures. Counting to one million gives students a real reason to use calculators to find averages and calculate different counting rates. So take a deep breath and join the counting fun!

COUNTING INVESTIGATION 1

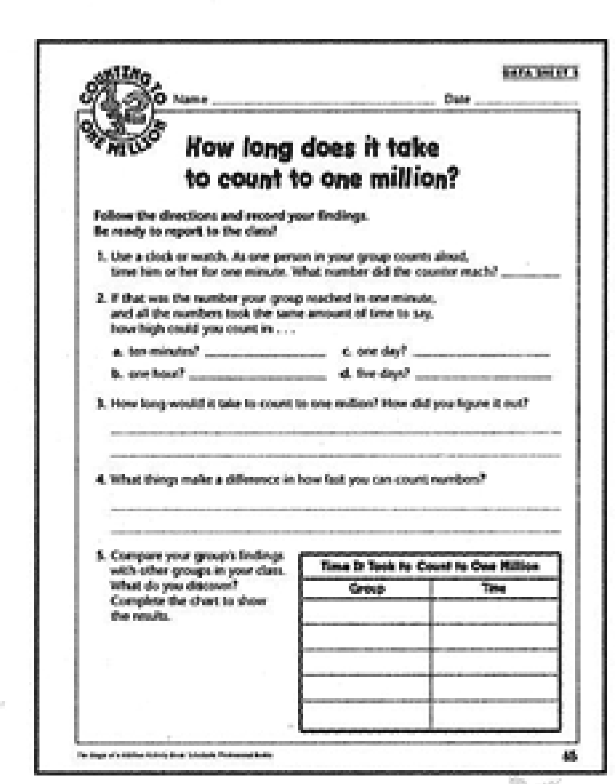
Clocking One Million

BIG IDEA: How long does it take to count to one million?

PROCESS SKILLS: counting, recording, explaining methods, developing mathematical strategies

What to Do

1. Ask students how long they think it will take to count to one million. If you've read David Schwartz's *How Much Is a Million?* ask students if they think 23 days, as reported in the book, is a reasonable answer.
2. Tell students you will assume that all the numbers between 1 and 1,000,000 take the same amount of time to say. Ask a volunteer to come to the front of the classroom and begin counting. After one minute has passed, write on the chalkboard the number the counter reached. Discuss the results with the class. What conclusions can they reach, with this sample, about counting to one million?
3. Invite students to make some estimates about how long it will take to reach one million, based on this one-minute counting sample. Ask them to suggest ways they could find an answer.
4. Let students try out their suggestions, working in pairs or small groups. If you wish, copy Data Sheet 3 on page 65 and have students use it to record their findings.
5. When all groups are finished, discuss their results. Are all answers within a similar range? If not, ask the class to decide why not. If students do not raise the idea themselves, you might want to point out that as numbers get bigger, they take longer to say. Can students figure out a way to account for this important fact?



Name ______ Date ______

How long does it take to count to one million?

Follow the directions and record your findings. Be ready to report to the class!

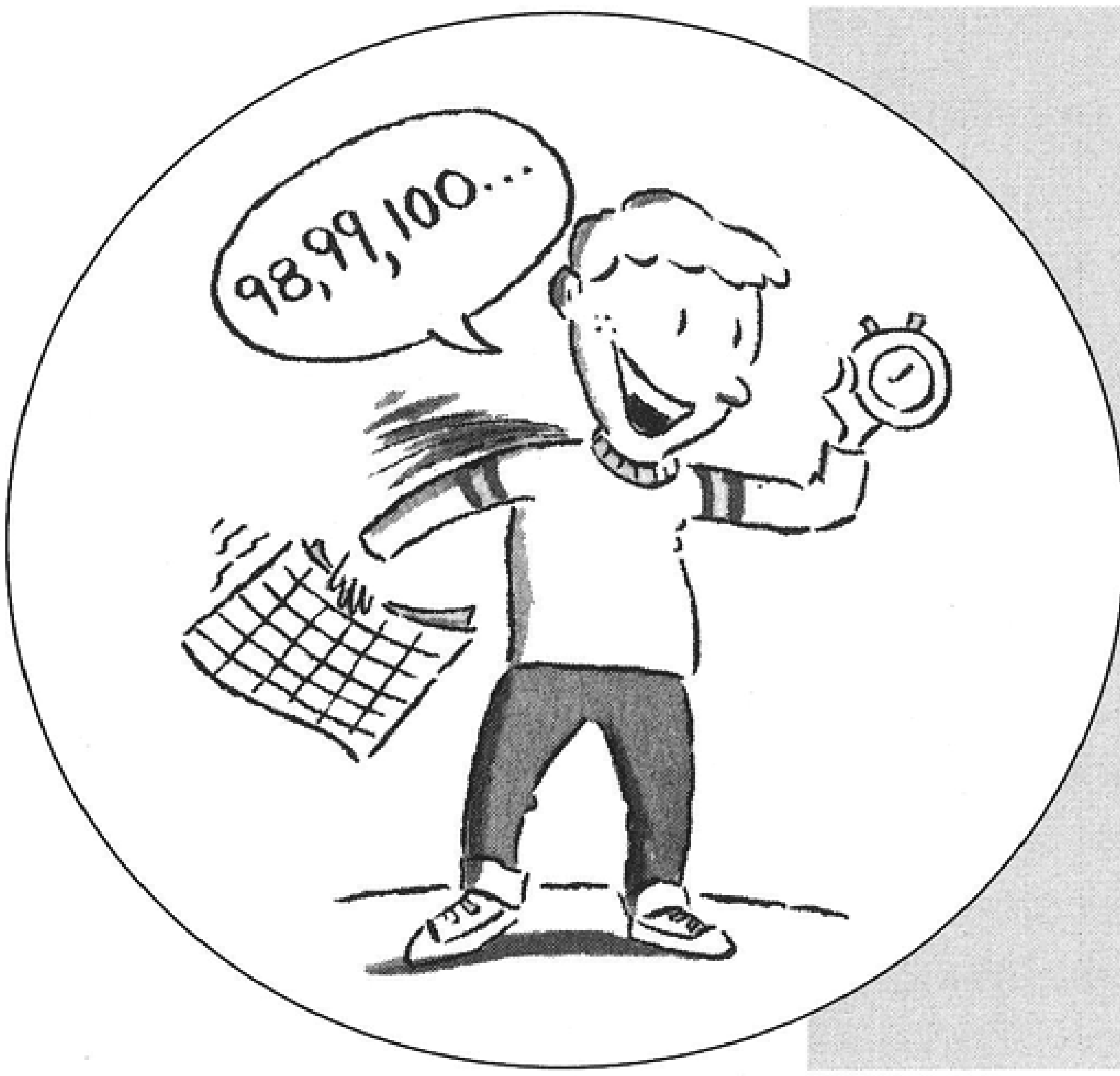
1. Use a clock or watch. As one person in your group counts aloud, time him or her for one minute. What number did the counter reach? ______
2. If that was the number your group reached in one minute, and all the numbers took the same amount of time to say, how high could you count in . . .
   a. ten minutes? ______ c. one day? ______
   b. one hour? ______ d. five days? ______
3. How long would it take to count to one million? How did you figure it out?
4. What things make a difference in how fast you can count numbers?
5. Compare your group's findings with other groups in your class. What do you discover? Complete the chart to show the results.

Time It Took to Count to One Million	
Group	Time

65

The Math Classroom in Action

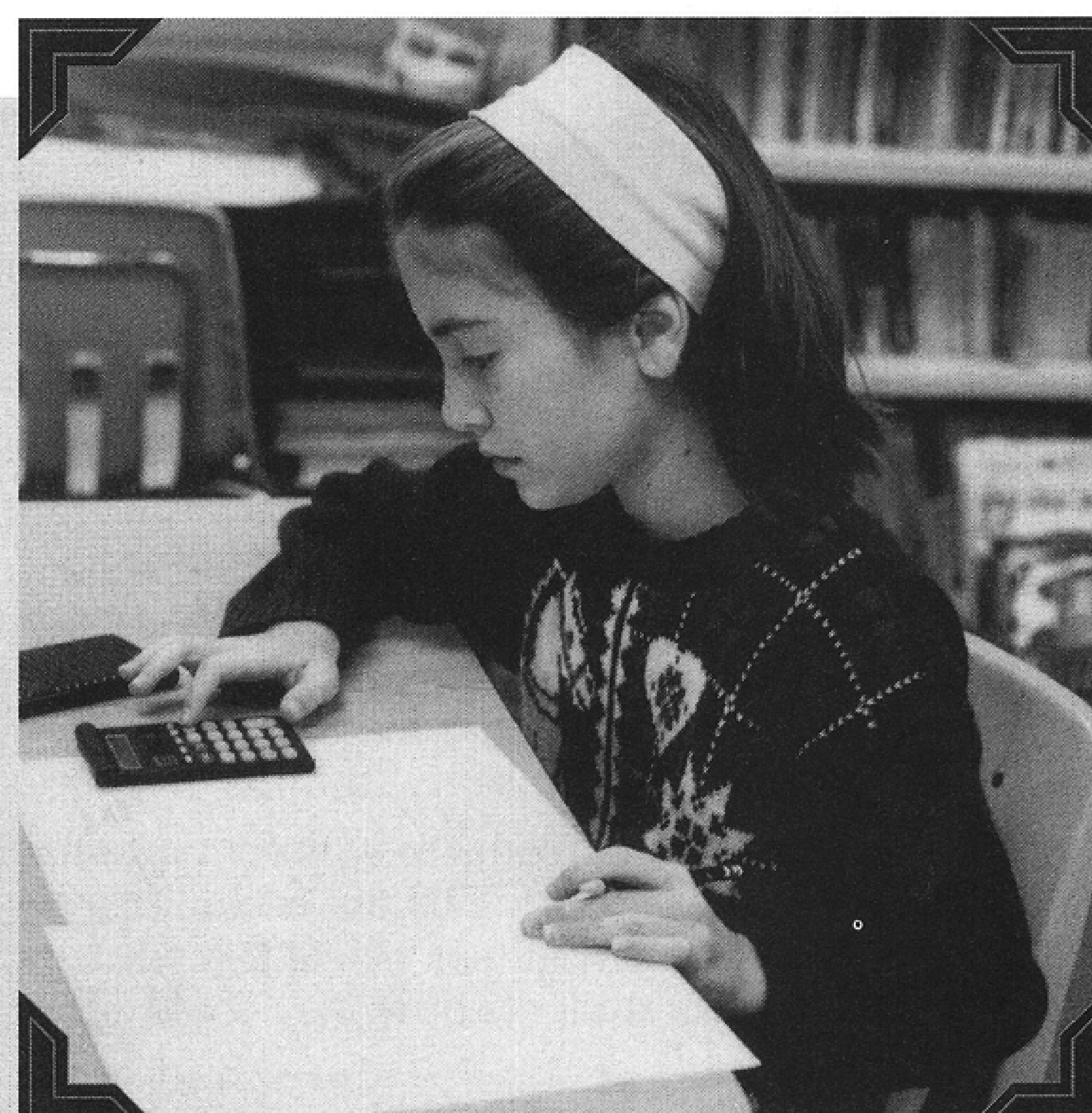
Clocking One Million



In one fifth-grade class, Philip counted to 194 in two minutes. He calculated his rate of counting: 194 ÷ 120 seconds = 1.6 seconds. Philip reasoned that it takes 1.6 seconds to say each number. He used that data to calculate the time it would take to count to one million. His conclusion (numbers are rounded):

$$1.6 \times 1{,}000{,}000 = 1{,}600{,}000 \text{ seconds}$$
$$1{,}600{,}000 \div 60 = 26{,}667 \text{ minutes}$$
$$26{,}667 \div 60 = 444 \text{ hours}$$
$$444 \div 24 = 19 \text{ days}$$

This would be a good solution to the problem if it took the same amount of time to count all the numbers. Although it does not, students often use this as an assumption.



Some fourth graders realized that larger numbers take longer to say than smaller ones. They decided to have different children start counting at different numbers. Then they counted for one minute and recorded their results in a chart. With their teacher's help, they used a calculator to see how far each person would have gotten in an hour (multiply by 60) and in a day (multiply that result by 24), if they continued at that same rate.

Some of the students' insights into factors affecting the counting time are:

- ★ Larger numbers take longer to say.
- ★ Some people count more quickly than others.
- ★ If you tried to just keep counting to reach one million, you would have to take breaks and it would take longer than the calculated time.

COUNTING INVESTIGATION 2

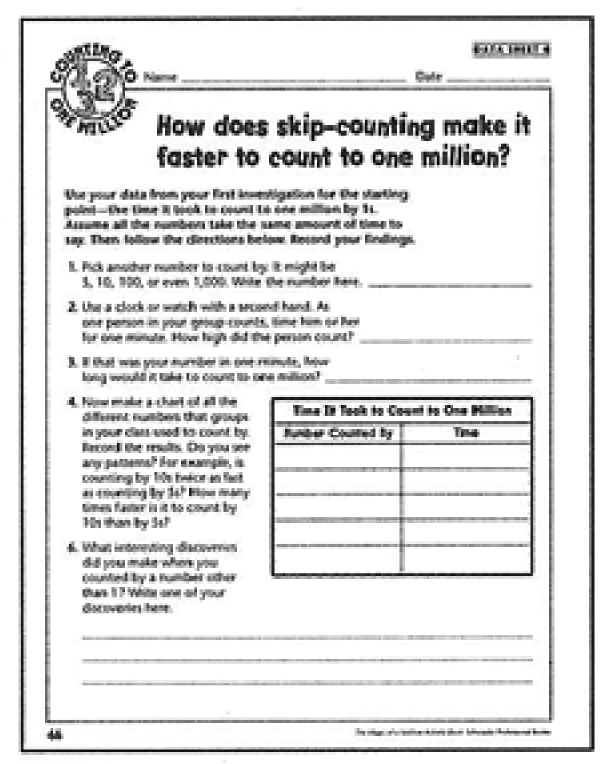
Double-Time

BIG IDEA: How does skip-counting change the time it takes to count to one million?

PROCESS SKILLS: counting by numbers other than 1, making predictions, recording, comparing results

What to Do

1. Ask students if they can think of some faster ways to count to one million. Some students will likely suggest trying to say numbers faster. Others will probably suggest skip-counting by 2s, 5s, 10s, 100s, or 1,000s. If a variety of numbers is suggested, ask children to predict which will be the fastest and why they think so. How much faster would it be than counting by 1s?
2. Divide the class into small groups. Give each group a number to count by, and challenge each group to figure out how long it will take to count to one million.
3. If you wish, copy and distribute Data Sheet 4 on page 66 and have students use it as they work. When all groups are finished, compile the results in a class chart such as the one shown below.


Name Date

How does skip-counting make it faster to count to one million?

Use your data from your first investigation for the starting point—the time it took to count to one million by 1s. Assume all the numbers take the same amount of time to say. Then follow the directions below. Record your findings.

1. Pick another number to count by. It might be 5, 10, 100, or even 1,000. Write the number here.
2. Use a clock or watch with a second hand. As one person in your group counts, time him or her for one minute. How high did the person count?
3. If that was your number in one minute, how long would it take to count to one million?
4. Now make a chart of all the different numbers that groups in your class used to count by. Record the results. Do you see any patterns? For example, is counting by 10s twice as fast as counting by 5s? How many times faster is it to count by 10s than by 5s?
5. What interesting discoveries did you make when you counted by a number other than 1? Write one of your discoveries here.

Time It Took to Count to One Million	
Number Counted By	Time

66

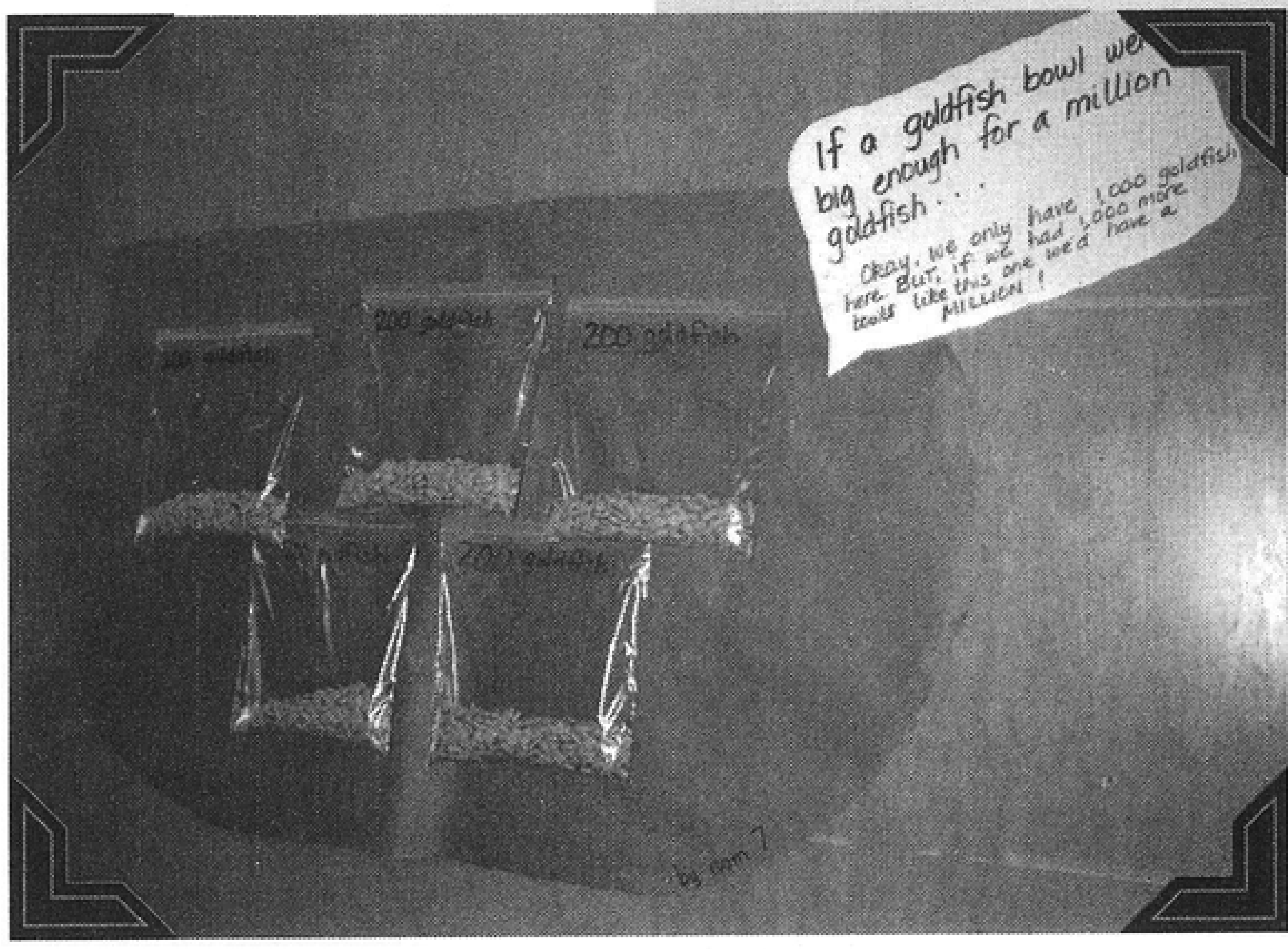
Number Counted By	Time It Takes to Reach One Million

Taking It Further

- ★ Ask students how they would figure out the number of tricycles it would take to have one million tricycle wheels. Could this question be answered by skip-counting? How about the number of octopuses to have one million octopus legs? Or soccer teams to have a million players?
- ★ Challenge students to look for patterns when they count. Is counting by 10s twice as fast as counting by 5s? Is counting by 8s four times as fast as counting by 2s? How many numbers would be counted if you counted to 1,000,000 by 2s, 3s, 7s, etc.? Ask students how they could figure that out.

The Math Classroom in Action

Double-Time



After their class had read David Schwartz's *How Much Is a Million?*, students at Laurel Wood Elementary School in Salinas, California, were inspired to investigate the size of bowl that would be needed to hold one million goldfish. What could be more natural than to use "real" goldfish! These students employed the idea of skip counting using a BIG number. As they bagged groups of 200 fish, they found that they needed 5,000 bags to count one million goldfish.

Some fourth grade students were interested in figuring out how long it would take to count to one million using a number greater than 1. The class figured that if they counted by 2s they should reach one million in half the time, or 11.5 days. When their teacher asked them how long it would take them to count to one million by 5s, Ben reasoned, "Well, it has to be less than 5 days because 5 times 5 is 25, and it's only 23 days. It must be between 4 and 5 days." Students used calculators and compiled this chart:

Counting by	Time to Reach One Million
1s	23 days
2s	11.5 days
3s	7.6666 days
4s	5.75 days
5s	4.6 days
6s	3.8333 days
7s	3.2857 days
8s	2.875 days
9s	2.5555 days
10s	2.3 days

One student looked at the sequence of numbers 1, 2, 4, 8, and 16, and found that each time a number is doubled, the amount of time to reach one million is cut in half. The moral of this investigation: If you want to finish a job in a hurry, work twice as fast and you'll be done in half the time!

COUNTING INVESTIGATION 3

Say it or Write It?

BIG IDEA: Is it faster to count aloud to one million or to write numbers from one to one million?

PROCESS SKILLS: counting, recording, finding an average, explaining methods

What to Do

1. Ask students which they think would take longer: to count aloud from one to one million or to write numbers from one to one million. As students respond, ask them to give reasons for their answers. Then invite them to try it out.
2. Have ten students write for one minute. Record the number each student reaches. It is likely that students will not all reach the same number after a minute of writing. This is a perfect time to introduce or reinforce the concept of averaging to determine a number that can be used as a benchmark. Show students how to add the 10 numbers and then divide the sum by 10 to find the average.
3. If you wish, copy and distribute Data Sheet 5 on page 67. Suggest that students use their benchmark number as the starting point. Then ask students to figure out how long it would take to reach one million by writing the numbers. They should use any method they like to accomplish this, including using a calculator. Remind students to complete the portions of the recording sheet that ask them to explain and evaluate their procedures.


Is it faster to count aloud to one million or to write numbers from one to one million?
1. Number reached in one minute:
2. Time to reach one million:
3. Here's how I got my answer:
4. I think my result is (accurate/not accurate) because

4. When everyone is finished, ask students to share their results. As different students present their findings, have them explain the methods they used. If results vary, ask students to discuss why this might happen. If two students have used the same method, do their answers agree? If not, whose answer is correct?

BIG Number Fact

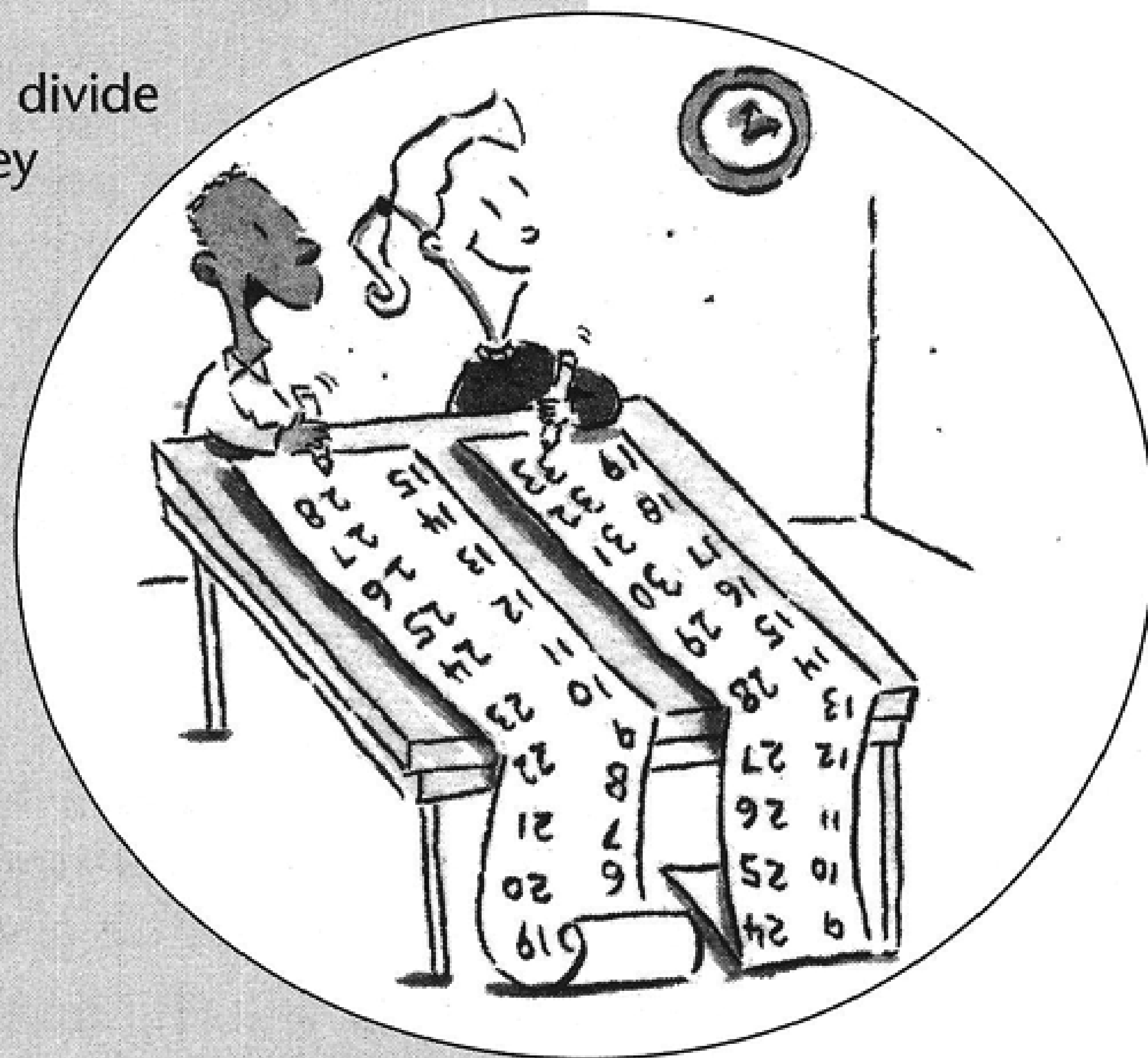
There are about 100 billion stars in the Milky Way galaxy. If you tried to count to 100 billion, it could take as long as 15,000 years.

The Math Classroom in Action

Say It or Write It?

When some sixth graders wanted to know if it would be faster to count or write to one million, one student suggested that they should start by having everyone write as fast as they could for one minute and see how far they got. The class agreed, and wrote furiously for one minute. Their ending numbers ranged from 41 to 67.

The students decided to add all the figures and divide by the number of students to find the average. They determined the average to be 53.



However, they decided that larger numbers would take longer to write than smaller numbers. They wanted to take that fact into account in their calculations. Instead of conducting additional tests for the larger numbers, which they could have done, they decided to estimate some of the other rates. They agreed on the following rates:

1- and 2-digit numbers	**53 per minute**
3-digit numbers	**35 per minute**
4-digit numbers	**27 per minute**
5-digit numbers	**21 per minute**
6-digit numbers	**18 per minute**
7-digit numbers	**only one number would have to be written.**

The students then added up these rates and divided by 5, rounding off their answer to get an average rate.

$53 + 35 + 27 + 21 + 18 + 1 = 155 \qquad 155 \div 5 = 30$

The students then calculated the time in the following manner:

30×60 minutes $= 1{,}800$ (the number reached in one hour)
$1{,}800 \times 24 = 43{,}200$ (the number reached in one day)
$1{,}000{,}000 \div 43{,}200 = 23$ (the number of days to reach 1,000,000)

Some interesting strategizing went into this solution, but unfortunately it is mathematically flawed. Ninety percent of the numbers between 1 and 1,000,000 are 6-digit numbers, but averaging the various rates gives equal weight to all of them. It would have been reasonable to use the 6-digit figure of 18 and throw the others away. Or students could have raised the figure a bit—perhaps to 19 or 20 per minute—to account for how much faster one can say the numbers between 1 and 100,000. The students in this class were amazed that their final answer matched the figure in *How Much Is a Million?* We think it's amazing, too, but we believe the close coincidence of the two figures is just that—a coincidence!

COUNTING INVESTIGATION 4

Faster Than a Speeding Calculator!

BIG IDEA: Is counting with a calculator faster than counting aloud?

PROCESS SKILLS: using a calculator, recording, explaining methods

What to Do

1. Invite students to suggest ways they could use a calculator to help them count to one million. Many creative suggestions will emerge. But if you want to stress strictly counting, you may need to demonstrate how to count with a calculator—by pressing 1 + 1 and then pressing "equals" again and again. Show students what the calculator records when you do this. If each student or small groups have calculators, invite them to try.
2. Ask a volunteer to use a calculator and to begin counting. After a minute, stop and record the result on the chalkboard. Compare the result with students' results when they counted aloud for one minute. Discuss with students: We know that numbers take longer to say as they get larger. But does it take longer to compute in this way on the calculator as the numbers get larger? Why or why not?
3. Invite students to predict how long it will take to reach one million using the calculator. If only one calculator is available, you can complete the activity as a class and compile the results together.
4. Discuss the results. Ask children to talk about what they discovered as advantages or disadvantages of using a calculator to do the counting. Was it more tiring to say numbers or punch calculator buttons? Was it more likely that you would lose track of counting by one method or the other? Ask students if they can think of any practical reason to use a calculator in this way to count to one million. (We can't!)

Taking It Further

★ Students might like to try setting up a computer program to count to one million. They can then compare that rate to the results of counting with a calculator. They might program the computer to count by 2s, 3s, 4s, and so on and again compare the results. You can also show students how they can use a calculator to count by 2s, 3s, or any number.

★ With your class, discuss the question: Is it ever necessary to count to one million? If so, when? How would you do it?

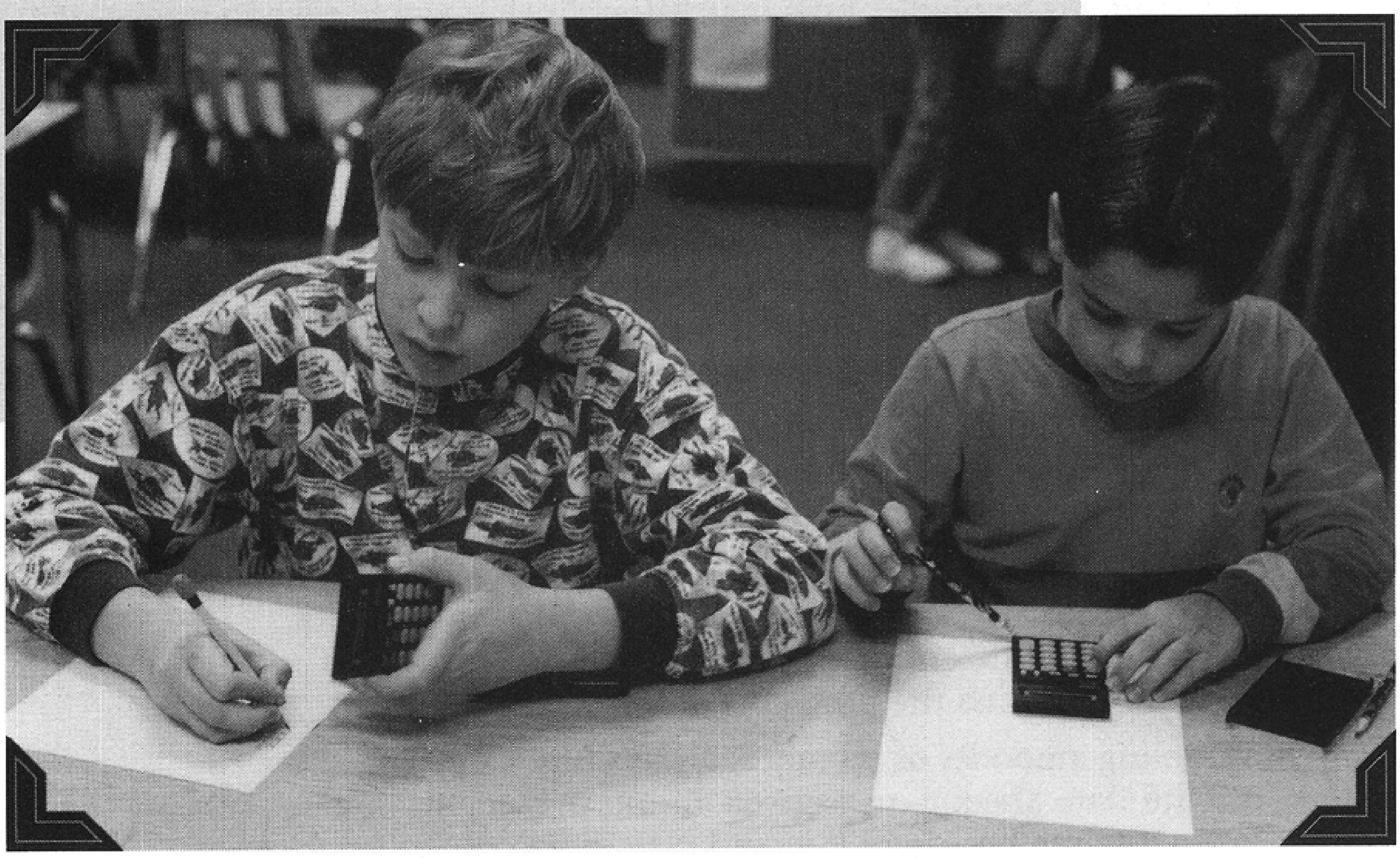
The Math Classroom in Action

Faster Than a Speeding Calculator!

Jordan and Adam decided to use the speed of a calculator to help them reach the one million mark in record time. Here's what Adam wrote about their procedures:

"Jordan and I took a calculator and pressed 1 + 1 and then kept pressing 'equals'. We timed it with a stopwatch. We would switch off. We reached 18,311 in 1 hour, 5 minutes and 57 seconds. We then figured that it would take 54.5 more hours. It would take 55.5 hours to get to 1,000,000 on a calculator."

(Although they didn't need to, since they were using a calculator, the boys decided to round their total to 18,000 and then divide 1,000,000 by that number to get 55.5 hours.)



Fourth grader Casey used the calculator to figure out how long it would take to count to one million by 2s, 3s, 4s, and so on. When she reached 11s, she found that it would take 2.0909 days. This led to a conversation with her teacher about rounding, since she wasn't sure how to round that number to the nearest tenth.

The calculator + curiosity + teacher's help = new learning for Casey!

COUNTING INVESTIGATION 5

Getting There in a Hurry

BIG IDEA: How long does it take to reach one million using doubling?

PROCESS SKILLS: counting, doubling and other geometric progressions, recording, explaining results

What to Do

1. Invite students to try doubling numbers. Work together at first, so they get an idea of how quickly doubling "adds up." For example:

   Start with 1. Double it. We get (2).
   Start with 2. Double it. We get (4).
   Start with 4. Double it. We get (8).
   Start with 8. Double it. We get (16).
   Start with 16. Double it. We get (32).

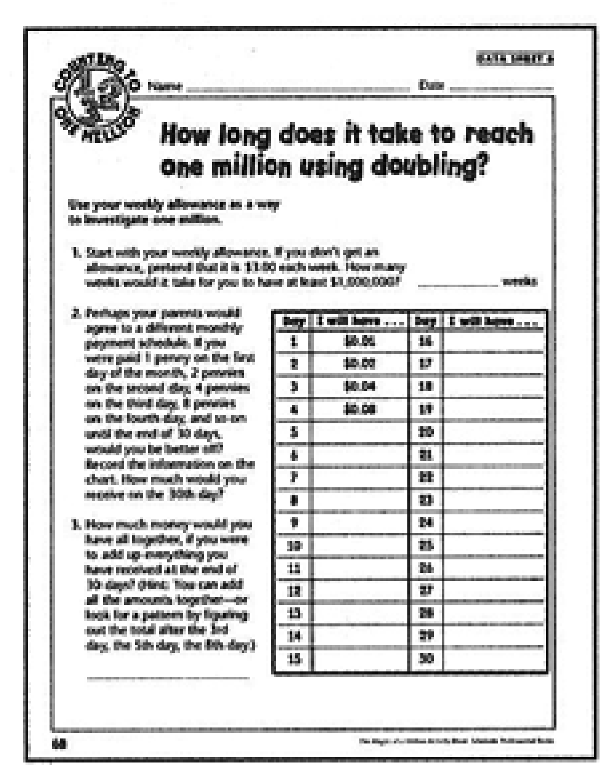
   Ask students to predict how many doublings it would take to reach 1,000,000. Record their guesses. Have students work in pairs or small groups to find out. They will undoubtedly be amazed to see that they'll reach 1,048,576 on the twentieth double.

2. Introduce the concept (not necessarily the term) of a geometric progression: multiplying by a certain constant each time; in this case, by 2. Give some real-life examples of geometric progressions and have students discover the implications. For example *The population of Nicaragua is about 4.5 million people. It will double every 20 years. In a lifetime of 60 years, what will happen to the population?*

   Present time . . . 4.5 million people
   In 20 years. . . . . (9 million)
   In 40 years. . . . . (18 million)
   In 60 years. . . . . (36 million)

   Using an almanac or other source of population statistics, have students determine population growths of various countries. Discuss the problems of this kind of growth.

3. Doubling amounts of money is always fun. Copy and distribute Data Sheet 6 on page 68 and have students complete it to see how long it would take to become a millionaire using a doubling payment schedule.


How long does it take to reach one million using doubling?

Taking It Further

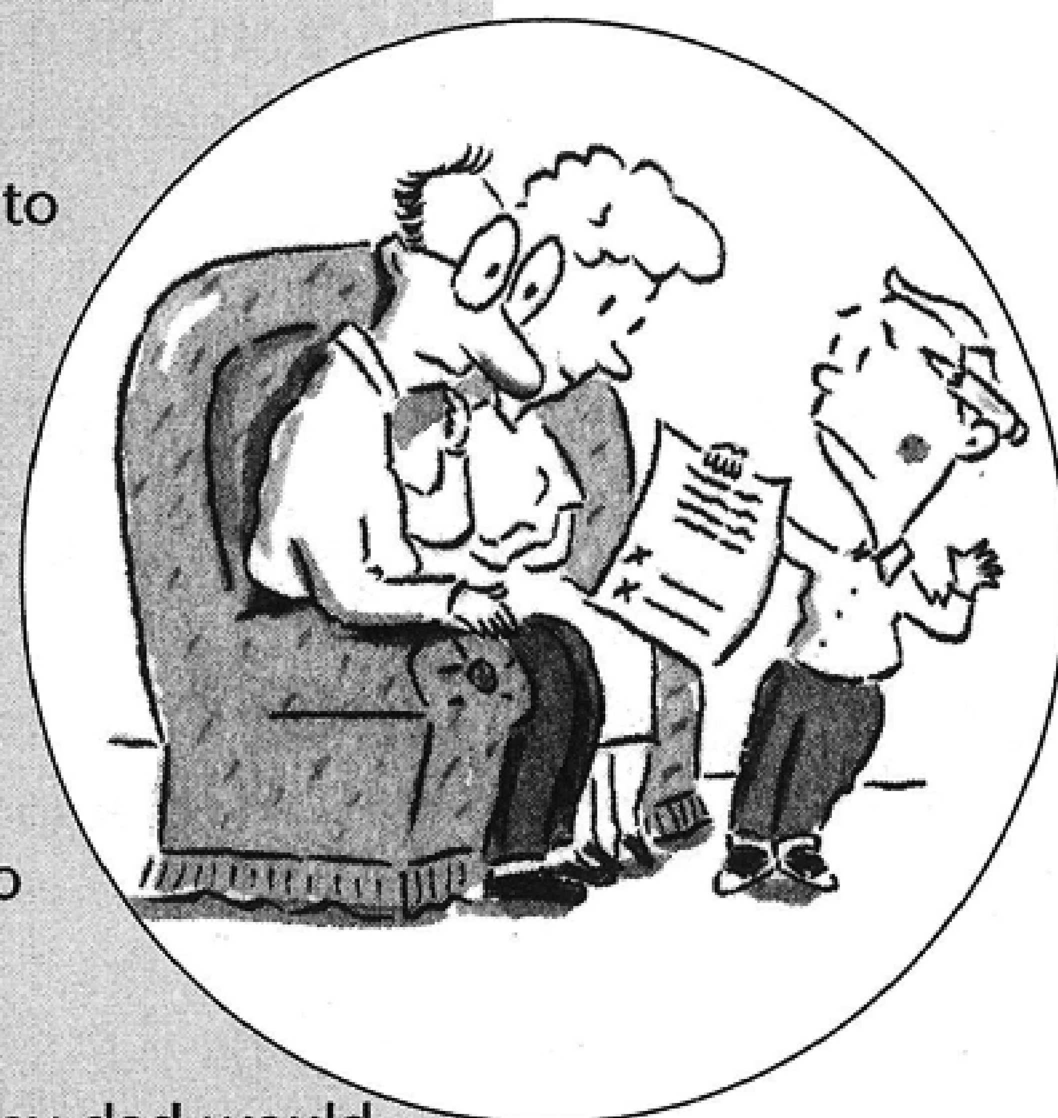
★ What happens when you triple your way toward one million? Challenge students to find out. If larger numbers produce more rapid expansion toward one million, ask students to investigate doubling numbers less than one, such as ½ or ¼.

★ Bacteria are single-celled organisms that reproduce by dividing in two. Suppose a certain kind of bacterium divides every 30 minutes. How long would it take before a single bacterial cell has become 1,000,000 bacteria?

The Math Classroom in Action

Getting There in a Hurry

One group of fifth graders attempted to "trick" some unsuspecting parents into the double-a-penny-each-day method of paying allowance. Some students even wrote official contracts for their parents to sign. Here are the results they reported:



"When we walked into the house I ran to get the contract. When I gave it to my mom she was just about to sign when my dad had to figure it out. I was frantically trying to wave him off. . . But it still didn't work. But wait—one more chance left—do your 'Sorry face'! Then my mom yells, 'Quit pouting!' Oh well. Don't ever try to bribe your parents because sooner or later they're going to say those two letters—NO!" —*Matt*

"I was thinking about what my dad would say. I thought he would say no. I wrote an interesting contract to try to get him in a good mood. Then I went to him and said, 'Hey Dad, you know, instead of paying me $3.75 for my allowance, pay me $.25 and double it each day for only two weeks.' And he said, 'Forget it. Do you know how much money that is?' I said, 'Yes sir.' And he pulled out a calculator and figured it out and was astonished." —*Tara*

"I approached my dad after we were done eating supper [and asked him about a doubling payment for my allowance]. He said, 'I guess I will.' Then I showed him how much money he would have to pay me. Then he said, 'There goes my tax money!'" —*Brian*